RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College under University of Calcutta)
B.A./B.Sc. SIXTH SEMESTER EXAMINATION, MAY 2014
THIRD YEAR
Date : 24/05/2014 **MATHEMATICS (Honours)**
Time : 11 am – 2 pm **Paper :** VIII Full Marks : 70

[Use a separate Answer book for each Group]

Group - A

[*Answer **any five** questions*]

1. If a variable system of coplanar forces have constant moments about two fixed points on the plane, prove that the resultant passes through another fixed point. [6]
2. State and prove the principle of virtual work for any system of coplanar forces acting on a rigid body. [6]
3. A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ, ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove that $\tan\phi = \frac{3}{8} + \tan\theta$. [6]
4. A square Lamina rests with its plane perpendicular to a smooth wall, one corner being attached to a point in the wall by a fine string of length equal to the side of the square. Find the position of equilibrium and show that it is stable. [6]
5. A force parallel to the axis of z acts at the point $(a,0,0)$ and an equal force perpendicular to the axis of z acts at the point $(-a,0,0)$. Show that the central axis of the system lies on the surface $z^2(x^2+y^2) = (x^2+y^2-ax)^2$. [6]
6. A perfectly rough plane is inclined at an angle α to the horizon. Show that the least eccentricity of the ellipse which can rest on the plane is $\sqrt{\frac{2\sin\alpha}{1+\sin\alpha}}$. [6]
7. A heavy uniform string rests on the upper surface of a rough vertical circle of radius a, and partly hangs vertically. If one end be at the highest point of the circle, show that the greatest length that can hang freely is $\frac{2a\mu + (\mu^2-1)ae^{\frac{\mu\pi}{2}}}{1+\mu^2}$, where μ is the coefficient of friction. [6]
8. Obtain the general Cartesian equations of equilibrium of a string under coplanar forces. Using it find the form of equilibrium of a string hanging under gravity when mass per unit length varies as the tension there. [6]

Group - B

[*Answer **any two** questions*]

9. a) What do you mean by assembly language? [3]
   b) Describe storage classes. [4]
   c) A C program contains the following declaration : [3]

```
static float table [2][3] = {
                             {1·1, 1·2, 1·3}
                             {2·1, 2·2, 2·3}
                             }
```

   i) What is the meaning of table?
   ii) What is the meaning of (table + 1)?

iii) What is the meaning of *(table +1)?
iv) What is the meaning of (*(table + 1)+1)?
v) What is the value of *(*(table+1)+1)?
vi) What is the value of *(*(table)+1)?

10. a) 50 floating-point data is stored in a data file stand.dat. Write a C program to evaluate the standard deviation of the stored data. [5]
b) Two integer type arrays contain two matrices, one of order 4×4 and the other of order 4×5, in a data file mat.dat. Write an efficient C program to determine the product of the two matrices and store the result in an output file. [5]

11. a) In a Boolean algebra $(B, +, \cdot, ')$, prove that for all $a, b, c \in B$, if $a+b = a+c$ and $a \cdot b = a \cdot c$ then $b = c$. [3]
b) Find the disjunctive normal form of the Boolean function f(x,y,z) such that f(x,y,z) = 1 if and only if two or more of the variables are 1. [3]
c) A committee consists of the President, Vice President, Secretary and Treasurer. A proposal is approved if and only if it receives a majority vote or the vote of the President plus one other member. Each member approves the proposal by pressing a button attached to their seats. Design a switching circuit controlled by the buttons which allows current to pass if an only if the proposal is approved. [4]

Group – C

[Answer **either Unit I or Unit – II**]

Unit - I

[*Answer* ***any two*** *questions*]

12. a) Prove that δ^i_j is a mixed tensor of rank two. [2]
b) Find g and g^{ij} corresponding to the metric $(ds)^2 = 3(dx^1)^2 + 2(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^3$. [4]
c) Show that $\frac{\partial g^{ik}}{\partial x^j} = -g^{hk}\begin{Bmatrix} i \\ hj \end{Bmatrix} - g^{hi}\begin{Bmatrix} k \\ hj \end{Bmatrix}$. [4]

13. a) The components of a contravariant tensor in the co-ordinate system x^i are $A^{11} = 4, A^{12} = A^{21} = 0$, $A^{22} = 7$. Find its components in the coordinate system $\bar{x}^i$, where $\bar{x}^1 = 4(x^1)^2 - 7(x^2)^2$, $\bar{x}^2 = 4x^1 - 5x^2$. [5]
b) If A_i are the components of a covariant vector, then show that $\frac{\partial A_i}{\partial x^j}$ are not the components of a tensor but $\frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$ are the components of a tensor. [5]

14. a) Prove that the fundamental tensor g_{ij} is a covariant tensor of order two. [4]
b) If a tensor A_{ijkl} is symmetric in the first two indices from the left and skew-symmetric in the second and fourth indices from the left, show that $A_{ijkl} = 0$. [3]
c) If A^{ijk} is a skew-symmetric tensor, show that $A^{ij\ell}_{,\ell} = \frac{1}{\sqrt{g}} \frac{\partial(\sqrt{g}A^{ijk})}{\partial x^k}$. [3]

Unit – II

[*Answer* ***any two*** *questions*]

15. a) Let $\alpha : I \to \mathbb{R}^2$ be a regular plane Curve. Let $[a, b] \subseteq I$ such that $\alpha(a) \neq \alpha(b)$. Prove that there exists some $t_0 \in (a, b)$ such that the tangent line of α at t_0 is parallel to the segment of the straight line joining $\alpha(a)$ with $\alpha(b)$.

b) Compute the curvature and torsion for the Curve $\alpha : \mathbb{R} \to \mathbb{R}^3$ given by

$$\alpha(t) = \left(a\cos\frac{t}{\sqrt{a^2+b^2}}, a\sin\frac{t}{\sqrt{a^2+b^2}}, \frac{bt}{\sqrt{a^2+b^2}} \right)$$ with $a>0$ and $b>0$.

c) Write down the Serret-Frenet equation for Space Curve. [4+3+3]

16. If $O \subseteq \mathbb{R}^3$ is open, $f : O \to \mathbb{R}$ is a differentiable function and a is a regular value of f belonging to its image

a) Prove that $S = f^{-1}(\{a\})$ is a surface.

b) Prove that $T_P S = \ker(df)_P$.

c) Let $f : S \to \mathbb{R}^n$ be a differentiable map where S is a surface and S is connected. If $(df)_P = 0 \; \forall p \in S$ then show that f is constant. [4+3+3]

17. a) Let S be a surface and $X : U \to \mathbb{R}^3$ be a parametrization of S. Then show that there exits unit normal field defined on the open set V = X(U).

b) Let S be the surface given by $S = \{(x, y, z) \in \mathbb{R}^3 \mid 2z = x^2 + y^2\}$.

Find the Gauss Curvature and the Mean Curvature of S. [4+(3+3)]